UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
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CALCULO APLICADO
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(Solución)

1.-A) a)

$f'(x)=\frac{-2x}{(1+x^2)^2} \Rightarrow f'(x)>0\ \forall x<0 \therefore f(x)\ creciente\ en\ (-\infty,0)\ \wedge f(x)\ decrece\ en\ (0,\infty)$

b) $f''(x)=\frac{6x^2-2}{(1+x^2)^3}\geq 0\ si\ x\geq\frac{1}{\sqrt{3}}$ ó $x\geq-\frac{1}{\sqrt{3}}: en\ x=\pm\frac{1}{\sqrt{3}}\ cambio\ de\ concavidad$

$$f(x)\ U\ en\ \left(-\infty,-\frac{1}{\sqrt{3}}\right)\bigcup\left(\frac{1}{\sqrt{3}},-\infty\right)\ y\ f(x)\bigcap en\ \left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$

c)$f'(x)\Rightarrow x=0\ \wedge f''(0)<0\ ; x_0=0\ pto\ de\ máximo\ (f(0)=1) y\ como\ f(1)=f(-1)=\frac{1}{2}<f(0)\Rightarrow \pm 1 ptos\ de\ mínimo$

d) $f(x)\equiv 0\ y\ par\ \forall\ x$

B) $f'(1)=-\frac{1}{2}\ \wedge f'(-1)=\frac{1}{2}\ \therefore y-1=-\frac{1}{2}(x-1)\wedge y-1=\frac{1}{2}(x+1) las\ tangentes$

$$igualando: x=0\Rightarrow se\ cortan\ en\ el\ eje\ OY$$

2.- a) $L'H\ \lim_{x\to\infty}\frac{3x^2}{e^x}=\lim_{x\to\infty}\frac{6x}{e^x}=\lim_{x\to\infty}\frac{6}{e^x}=0$

b) $\lim_{x\to\infty}\frac{2x^2}{x^2-1}=\lim_{x\to\infty}\frac{2}{1-\frac{1}{x^2}}=2$

c) $\lim_{x\to 0^+} e^{\ln(tg\,x)^{sen\,x}}\Leftrightarrow \lim\limits_{x\to 0^+}\ln(tg\,x)^{sen\,x}=\lim\limits_{x\to 0^+}\frac{\ln(tg\,x)}{cosec\,x}\left(\frac{\infty}{\infty}\right)=\lim\limits_{x\to 0^+}\frac{\frac{sec^2x}{tg\,x}}{cosec\,x\,cotg\,x}=$

$\lim\limits_{x\to 0^+}\frac{sec^2x}{cosec\,x}=\lim\limits_{x\to 0^+}\frac{sen\,x}{cos^2x}=0\ \therefore \lim f(x)=e^0=1$

3.- $I_m=\int x^m e^{x^2}dx: por\ partes:$

$u=x^{m-1}\Rightarrow du=(m-1)x^{m-2}dx$

$dv=xe^{x^2}dx\Rightarrow v=\frac{1}{2}e^{x^2}$

$$I_m=\frac{x^{m-1}e^{x^2}}{2}-\frac{(m-1)}{2}\int x^{m-2}e^{x^2}dx=\frac{x^{m-1}e^{x^2}}{2}-\frac{(m-1)}{2}I_{m-2}$$

$$I_5 = \frac{x^4 e^{x^2}}{2} - 2I_3 = \frac{x^4 e^{x^2}}{2} - 2\left(\frac{x^2 e^{x^2}}{2} - I_1\right) = \frac{x^4 e^{x^2}}{2} - x^2 e^{x^2} + 2\int x e^{x^2} dx$$

$$I_5 = \left(\frac{x^4 e^{x^2}}{2} - x^2 e^{x^2} + e^{x^2}\right)_0^1 = \frac{e}{2} - 1$$

4.- a) $Sustituir: u = \ln x \quad du = \frac{dx}{x} \Rightarrow I = \int \frac{du}{4+9u^2} = \frac{1}{4}\int \frac{du}{1+\left(\frac{3}{2}u\right)^2}$

b) $Fracciones\ parciales: I = \frac{5}{27}\int \frac{dx}{x-3} - \frac{1}{27}\int \frac{5x-24}{x^2+3x+9} dx = \frac{5}{27}\ln|x-3| - \frac{1}{27}\ I_2$

$$I_2 = 5\int \frac{x - \frac{24}{5}}{x^2+3x+9} = \frac{5}{2}\int \frac{2x - \frac{48}{5}}{x^2+3x+9} = \frac{5}{2}\int \frac{2x + 3 - \frac{63}{5}}{x^2+3x+9} = \frac{5}{2}\left(\ln(x^2+3x+9) - \frac{63}{2} I_3\right)$$

$$I_3 = \int \frac{dx}{x^2+3x+9} = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 + \frac{27}{4}} = \int \frac{du}{u^2 + \frac{27}{4}}$$