Solución Pep 1 2006

1.- a) Como: $(a+b) \geq 2\sqrt{ab}$

$$(b+c) \geq 2\sqrt{bc} \Rightarrow (a+b)(b+c) \geq 4\mathrm{b}\sqrt{ac} \geq \mathrm{b}\sqrt{ac}$$

b) $(a+b)^2 \geq 4ab \geq ab$

$(b+c)^2 \geq 4bc \geq bc$

$(c+a)^2 \geq 4ac \geq ac \Rightarrow (a+b)^2(b+c)^2\ (c+a)^2 \geq a^2b^2c^2$

2.- Condición D: $4(m-3)^2 + 4(1-3m)(m-1) \leq 0 \Rightarrow 1-3m < 0 \text{ ó } m > {}^{1}/_{3}$

$$m^2 - 6m + 9 + 4m - 3m^2 - 1 \leq 0 \Leftrightarrow -2m^2 - 2m + 8 \leq 0 \text{ ó } m^2 + m - 4 \geq 0 \quad Si\ m^2 + m - 4 = 0 \Rightarrow m = \frac{-1 \pm \sqrt{17}}{2}$$

$$\therefore m \in \left(-\infty, \frac{-1-\sqrt{17}}{2}\right) \cup \left(\frac{-1+\sqrt{17}}{2}, \infty\right) \cap \left(\frac{1}{3}, \infty\right) \equiv \left(\frac{-1+\sqrt{17}}{2}, \infty\right)$$

Conclusión $4(m-3)^2 + 4(1-3m)(m-1) < 0 \quad 1-3m < 0$

3.- Sí $\sqrt{x^2-4} \leq 2; condición\ x^2 - 4 \geq 0 : \ (x \geq 2 \ \vee x \leq -2)$

$$\therefore\ x^2 - 4 < 4 \ \Rightarrow \ x^2 < 8 \Rightarrow \left(-2\sqrt{2} < x < 2\sqrt{2}\right) pero\ (x \geq 2 \ \vee x \leq -2)$$

$$A = \left\{x \middle| x \in \left(-2\sqrt{2}, -2\right] \cup \left[2, 2\sqrt{2}\right)\right\} Sup\ A = 2\sqrt{2} \quad , Inf\ A = -2\sqrt{2}$$

4.- a) $x^5 + 2x^4 - x - 2 > 0 \ \Leftrightarrow \ x^4(x+2) - (x+2) > 0 \Leftrightarrow (x^4 - 1)(x+2) > 0 \ \Rightarrow$

$(x+1)(x-1)(x+2)(x^2+1) > 0 \ \Leftrightarrow \ (x+1)(x-1)(x+2) > 0 : pues\ \ x^2 + 1 \geq 0\ \forall x.$

S=(-2,-2)U(1, ∞)

b) i) $|3x+1|$<2 $\Leftrightarrow -2 < 3x + 1 < 2 \Rightarrow -3 < 3x < 1 \Rightarrow -1 < x < {}^{1}/_{3}$

ii) $|\ x^2 + x - 6|$>2 $\Leftrightarrow x^2 + x - 6 > 2\ \vee\ x^2 + x - 6 < -2 \Leftrightarrow$

$(x^2 + x - 8 > 0) \vee (x^2 + x - 4 < 0)$

$x^2 + x - 8 = 0 \Rightarrow\ x = \dfrac{-1 \pm \sqrt{33}}{2}$

$x^2 + x + 4 = 0 \Rightarrow\ x = \dfrac{-1 \pm \sqrt{17}}{2}$

S_i= (-1, ${}^{1}/_{3}$)

$S_{ii}= \left(\frac{-1+\sqrt{33}}{2}, \infty\right) \cup \left(-\infty, \frac{-1-\sqrt{33}}{2}\right) \cup \left(\frac{-1-\sqrt{17}}{2}, \frac{-1+\sqrt{17}}{2}\right)$

$S_T=S_i \cap S_{ii}=(-1, {}^{1}/_{3})$