UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
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CALCULO APLICADO
PRUEBA Nº 4
(Solución)

1.

a) Plano tangente:

$$(x-x_0)F_x(P_0)+(y-y_0)F_y(P_0)+(z-z_0)F_z(P_0)=0$$

$$F_x(P_0)=2x_0; F_y(P_0)=-4y_0; F_z(P_0)=-8z_0 \Rightarrow$$

$\vec{M}=(2x_0,-4y_0,-8z_0)$; Condición: vectores normales paralelos:

$$(2x_0,-4y_0,-8z_0)=\lambda(4,-2,4)\Rightarrow x_0=2\lambda; y_0=\frac{\lambda}{2}; z_0=\frac{-\lambda}{2}$$

P0 está en la superficie $\Leftrightarrow (2\lambda)^2-2\left(\frac{\lambda}{2}\right)^2-4\left(-\frac{\lambda}{2}\right)^2=16\Rightarrow$

$$4\lambda^2-\frac{\lambda^2}{2}-\lambda^2=16:\frac{5\lambda^2}{2}=16\Rightarrow\lambda=\pm\frac{4\sqrt{2}}{\sqrt{5}}\Rightarrow$$

$$P_0\left(\frac{8\sqrt{2}}{\sqrt{5}},\frac{2\sqrt{2}}{\sqrt{5}},-\frac{2\sqrt{2}}{\sqrt{5}}\right)\ ;\ P_1\left(-\frac{8\sqrt{2}}{\sqrt{5}},-\frac{2\sqrt{2}}{\sqrt{5}},\frac{2\sqrt{2}}{\sqrt{5}}\right)$$

a.1) $\left(x-\frac{8\sqrt{2}}{\sqrt{5}}\right)\frac{16\sqrt{2}}{\sqrt{5}}+\left(y-\frac{2\sqrt{2}}{\sqrt{5}}\right)\left(-\frac{8\sqrt{2}}{\sqrt{5}}\right)+\left(z+\frac{2\sqrt{2}}{\sqrt{5}}\right)\left(-\frac{16\sqrt{2}}{\sqrt{5}}\right)=0\ \left|\frac{\sqrt{5}}{8\sqrt{2}}\right.$

$$\left(x-\frac{8\sqrt{2}}{\sqrt{5}}\right)2+\left(y-\frac{2\sqrt{2}}{\sqrt{5}}\right)(-1)+\left(z-\frac{2\sqrt{2}}{\sqrt{5}}\right)(-2)=0$$

$$2x-y-2z=\frac{\sqrt{2}}{\sqrt{5}}(16-2-4)=\frac{10\sqrt{2}}{\sqrt{5}}=2\sqrt{10}$$

a.2) $\left(x+\frac{8\sqrt{2}}{\sqrt{5}}\right)\frac{16\sqrt{2}}{\sqrt{5}}+\left(y+\frac{2\sqrt{2}}{\sqrt{5}}\right)\left(-\frac{8\sqrt{2}}{\sqrt{5}}\right)+\left(z-\frac{2\sqrt{2}}{\sqrt{5}}\right)\left(-\frac{16\sqrt{2}}{\sqrt{5}}\right)=0\ \left|\frac{\sqrt{5}}{8\sqrt{2}}\right.$

$$\left(x+\frac{8\sqrt{2}}{\sqrt{5}}\right)2+\left(y+\frac{2\sqrt{2}}{\sqrt{5}}\right)(-1)+\left(z+\frac{2\sqrt{2}}{\sqrt{5}}\right)(-2)=0$$

$$2x-y-2z=\frac{\sqrt{2}}{\sqrt{5}}(-16+2+4)=-\frac{10\sqrt{2}}{\sqrt{5}}=-2\sqrt{10}$$

2.

$f_x = 4x^3 - 2x = 0 \quad \rightarrow 2x(2x^2 - 1) = 0 \Rightarrow x = 0; x = \pm\frac{\sqrt{2}}{2}$

$f_y = -3y^2 + 1 = 0 \quad \rightarrow y = \pm\frac{\sqrt{3}}{3}$

$P_0\left(0, \frac{\sqrt{3}}{3}\right)$; $P_1\left(0, -\frac{\sqrt{3}}{3}\right)$; $P_2\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{3}\right)$; $P_3\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{3}\right)$; $P_4\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{3}\right)$;
$P_5\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{3}\right)$

Estudio del Hessiano:

$f_{xx} = 12x^2 - 2 \quad ; \quad f_{yy} = -6y \quad ; \quad f_{xy} = 0$

$$H = \begin{vmatrix} (12x^2 - 2) & 0 \\ 0 & -6y \end{vmatrix} = 12y(6x^2 - 1)$$

$H(P_0) = -4\sqrt{3} < 0$	: Pto. Silla
$H(P_1) = 4\sqrt{3} > 0;\ f_{xx} < 0$	: Pto. de Máximo
$H(P_2) = 8\sqrt{3} > 0;\ f_{xx} > 0$	: Pto. de Mínimo
$H(P_3) = 8\sqrt{3} > 0;\ f_{xx} > 0$	: Pto. de Mínimo
$H(P_4) = -8\sqrt{3} < 0$	: Pto. Silla
$H(P_5) = -8\sqrt{3} < 0$	: Pto. Silla

3.

a) $\nabla f = (f_x(P_0), f_y(P_0), f_z(P_0)) = \left(\frac{(yz)^{1/2}}{\sqrt{x}}, \frac{(xz)^{1/2}}{\sqrt{y}}, \frac{(xy)^{1/2}}{\sqrt{z}}\right)_{P_0} = \left(\frac{(12)^{1/2}}{\sqrt{3}}, \frac{(9)^{1/2}}{\sqrt{4}}, \frac{(12)^{1/2}}{\sqrt{3}}\right)$
$= \left(2, \frac{3}{2}, 2\right)$

b) $\left.\frac{\partial f}{\partial \hat{a}}\right|_{máx} = \|\nabla f\| = \sqrt{4 + \frac{9}{4} + 4} = \frac{1}{2}\sqrt{41}$

4.

Usando coordenadas polares:

$$I = \int_0^{2\pi}\int_1^2 \frac{\rho^2 sen\theta \cos\theta}{\rho^2} e^{\left(-\rho^2\right)} \rho d\rho d\theta$$

$$I = \int_0^{2\pi} sen\theta \cos\theta \left(-\frac{e^{\rho^2}}{2}\right)\Bigg|_1^2 d\theta = \frac{1}{2}\left(e^{-1} - e^{-4}\right)\int_0^{2\pi} sen\theta \cos\theta d\theta$$

$$I = \frac{1}{2}\left(e^{-1} - e^{-4}\right)\frac{sen^2\theta}{2}\Bigg|_0^{2\pi} = 0$$