UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
Prof: Jorge Inostroza L - Coordinador.

CALCULO APLICADO
EXAMEN Nº 2
(Solución)

1. Plano Tangente:

$$(x-x_0)F_x(P_0)+(y-y_0)F_y(P_0)+(z-z_0)F_z(P_0)=0$$

$P_0(-1,2,1)$:

$F_x(P_0)= yz-4z^3 = -2$

$F_y(P_0)= xz-6y^2 = -25$

$F_z(P_0)= xy-12xz^2 = 10$

$$(x+1)(-2)+(y-2)(-25)+(z-1)(10)=0$$

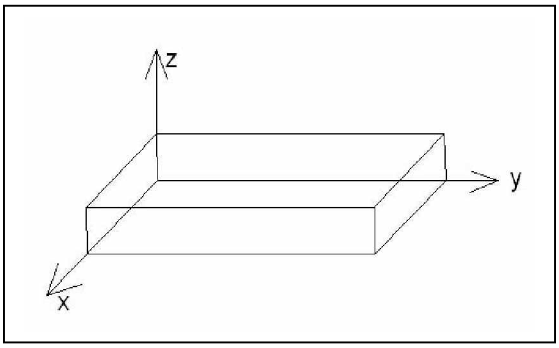
$$-2x-25y+10z=-38$$

Recta normal:

$$\frac{x-x_0}{F_x(P_0)}=\frac{y-y_0}{F_y(P_0)}=\frac{z-z_0}{F_z(P_0)}$$

$$\frac{x+1}{-2}=\frac{y-2}{-25}=\frac{z-1}{10}$$

2.



$V = xyz = 60$
$C(x,y,z) = 2xz + 2yz + 3xy + 2xy$
$L: 2xz + 2yz + 5xy + \lambda(xyz - 60)$

$$\left.\begin{array}{l} L_x: 2z + 5y + \lambda yz = 0 \\ \\ L_y: 2z + 5x + \lambda xz = 0 \\ \\ L_z: 2x + 2y + \lambda xy = 0 \end{array}\right\} \quad \begin{array}{l} \Rightarrow x = y \\ 4x + \lambda x^2 = 0 \\ x \neq 0 \end{array}$$

$$\left.\begin{array}{l} 2z + 5x = -\lambda xz \\ 4x = -\lambda x^2 \end{array}\right\} \quad \frac{2z+5x}{4x} = \frac{z}{x} \Rightarrow 2z + 5x = 4z$$

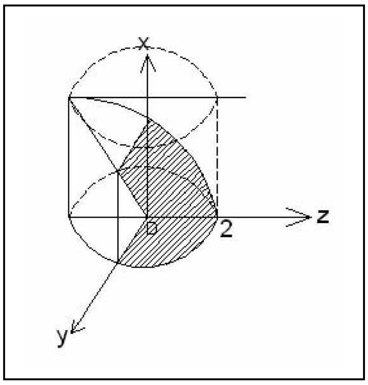
$$\frac{5}{2}x = z \quad \wedge \quad x = y \; ; \quad x = x$$

$$\frac{5}{2}xxx = 60 \Rightarrow x^3 = 24 \Rightarrow x = y = \sqrt[3]{24} = 2\sqrt[3]{3}$$

$$z = \frac{5 \cdot 2\sqrt[3]{3}}{2} = 5\sqrt[3]{3}$$

Este valor extremo corresponde aun cierto mínimo pues sería máximo si la base es infinitamente grande al igual que la tapa.

3.



$$V = \iint_{D_{yz}} x(yz)dA = \iint_D 2y\,dA \quad \text{Polares}$$

$$V = 2\int_0^{\pi/2}\int_0^2 \rho \, sen\theta \, \rho \, d\rho \, d\theta = 2\int_0^{\pi/2} \left(\frac{\rho^3}{3}\right)\Big|_0^2 sen\theta \, d\theta$$

$$V = \frac{16}{3}\int_0^{\pi/2} sen\theta \, d\theta = -\frac{16}{3}\cos\theta \Big|_0^{\pi/2} = \frac{16}{3}$$

4.

$$\frac{\partial}{\partial y}\left(5x^4y^4\right)=\frac{\partial}{\partial x}\left(4x^5y^3+1\right)=20x^4y^3$$

Luego es un campo gradiente; Calculando el potencial:

$$\frac{\partial\phi}{\partial x}=5x^4y^4\bigg| \int dx \Rightarrow \phi = x^5y^4+\kappa(y)\bigg| \frac{\partial}{\partial y}$$

$$\frac{\partial\phi}{\partial y}=4x^5y^3+\kappa`(`y)\equiv 4x^5y^3+1 \qquad \therefore \kappa`(y)=1 \quad \kappa(y)=y$$

$$\Rightarrow \phi(x,y)=x^5y^4+y$$

$$W=\int_C F(x,y)dp=\phi(-1,1)-\phi(0,0)=0$$

5.

a) $$\sum\frac{n!}{1\cdot3\cdot5\cdot7\cdots(2n-1)}=\sum\frac{n!\cdot 2\cdot4\cdot6\cdot2n}{(2n-1)!}=\sum\frac{n!\cdot2^n\cdot n!}{(2n-1)!}=\sum\frac{2^n(n!)^2}{(2n-1)!}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{2^{n+1}\cdot((n+1)!)^2}{(2n+1)!}\cdot\frac{(2n+1)!}{2^n(n!)^2}=\lim_{n\to\infty}\frac{2(n+1)^2}{2n(2n+1)}$$

$$=\lim_{n\to\infty}\frac{n^2+2n+1}{2n^2+n}=\lim_{n\to\infty}\frac{1+\frac{2}{n}+\frac{1}{n^2}}{2+\frac{1}{n}}=\frac{1}{2}<1$$

¡Converge!

b)

$$\sum(-1)^n\frac{1}{n^2}$$; Serie alternada y como:

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{1}{n^2}=0$$

La serie converge por criterio de Leibnitz.