UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
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CALCULO APLICADO
EXAMEN Nº 1
(Solución)

a) $\dfrac{\partial T}{\partial \hat{a}}(P_0) = \dfrac{\partial T}{\partial x}(P_0)a_1 + \dfrac{\partial T}{\partial y}(P_0)a_2 + \dfrac{\partial T}{\partial z}(P_0)a_3$

$\vec{a} = Q - P = (1,2,1)$; $\hat{a} = \dfrac{1}{\sqrt{6}}(1,2,1)$

$\dfrac{\partial T}{\partial x} = (-2x)T\Big|_{P_0} = -4T(P_0)$; $T(P_0) = 200e^{-43}$

$\dfrac{\partial T}{\partial y} = (-6y)T\Big|_{P_0} = 6T(P_0)$

$\dfrac{\partial T}{\partial z} = (-18z)T\Big|_{P_0} = -36T(P_0)$

$\dfrac{\partial T}{\partial \hat{a}}(P_0) = 200e^{-43}(-4+6-36) = \dfrac{200}{e^{43}\sqrt{6}}(-28)$

b) $\vec{a} = \nabla T(P_0) = (-4,6,-36)T(P_0)$: $\hat{a} = \dfrac{(2,3,-18)}{\sqrt{376}}$

$\dfrac{\partial T}{\partial \hat{a}}\Big|_{\substack{P_0 \\ máx}} = \|\nabla T\| = 2T(P_0)\sqrt{376}$

2.

a) Plano tangente:

$(x - x_0)2x_0 + (y - y_0)2y_0 + (z - z_0)(-2z_0) = 0$

P(0,0,0) está en el plano $\Leftrightarrow (0 - x_0)2x_0 + (0 - y_0)2y_0 + (0 - z_0)(-2z_0) = 0$

$-2x_0^2 - 2y_0^2 + 2z_0^2 = 0$

$x_0^2 + y_0^2 = z_0^2$ ¡Sí!

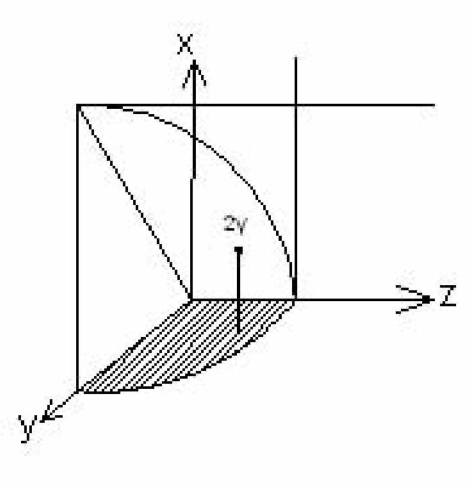
b) Recta normal:

$$\frac{x-x_0}{2x_0}=\frac{y-y_0}{2y_0}=\frac{z-z_0}{-2z_0} \quad ; \text{ Si } x=y=0$$

$$-\frac{1}{2}=\frac{z-z_0}{-2z_0} \Rightarrow z=2z_0$$

$\therefore$ Está al doble de la altura de P_0

3.



$$V=\iint_D 2y\,dA=\int_0^{\pi/2}\int_0^2 2\rho sen\theta \rho d\rho d\theta$$

$$V=\frac{2}{3}\int_0^{\pi/2}\rho^3\Big|_0^2 sen\theta d\theta=-\frac{16}{3}\cos\theta\Big|_0^{\pi/2}=\frac{16}{3}$$

4.

$$\frac{1}{R}=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{2^{n+1}(n+3)}{2^n(n+4)}=2 \qquad \therefore R=\frac{1}{2}$$

$$\therefore -\frac{1}{2}<x-3<\frac{1}{2}$$

a) si $x-3=-\frac{1}{2}\Rightarrow\sum(-1)^n\frac{1}{n+3}$ Converge Leibnitz

si $x-3=\frac{1}{2}\Rightarrow\sum\frac{1}{n+3}$ Diverge (Compara con $\sum\frac{1}{n}$)

$$\therefore -\frac{1}{2}\le x-3<\frac{1}{2}$$