UNIVERSIDAD DE SANTIAGO DE CHILE
DEPARTAMENTO DE MATEMÁTICA Y C.C
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CALCULO APLICADO
EXAMEN Nº 2
(Solución)

1.

Si

$x \geq 3 \Rightarrow -10 < (2 + x + 2(x-3)) < 10 \Leftrightarrow -6 < 3x < 14$

$\Leftrightarrow -2 < x < \frac{14}{3} \therefore 3 \leq x < \frac{14}{3}$

Si

$x \leq 3 \Rightarrow -10 < (2 + x - 2(x-3)) < 10 \Leftrightarrow -18 < -x < 2 \Leftrightarrow -2 < x < 18$

$\therefore S_b : -2 < x \leq 3 \therefore S_T = (-2, 14/3) \therefore$

$Sup : 14/3$

$Inf : -2.$

2.

$$I_n = \frac{1}{2}\int x^{n-1} 2xe^{x^2}$$

$$u = x^{n-1} \Rightarrow du = (n-1)x^{n-2}dx$$

$$dv = 2xe^{x^2} \Rightarrow v = e^{x^2}$$

$$I_n = \frac{1}{2}(x^{n-1}e^{x^2} - (n-1)\int x^{n-2}e^{x^2}dx) \therefore$$

$$I_n = \frac{1}{2}(x^{n-1}e^{x^2} - (n-1)I_{n-2})$$

$$I_5 = \frac{1}{2}(x^{n-1}e^{x^2} - 4\int x^3 e^{x^2}dx) \Big|_0^1 = \frac{1}{2}(e - 4 \cdot \frac{1}{2}(x^2 e^{x^2} - 2\int xe^{x^2}dx \Big|_0^1$$

$$I_5 = \frac{1}{2}(e - 2e + 4\int_0^1 xe^{x^2}dx)$$

$$I_5 = \frac{1}{2}(-e + 2e^{x^2}\Big|_0^1) = \frac{1}{2}(e-2).-$$

3.

Vector normal en P_o: $(2x_o, -4y_o, -8z_o) = \nabla F$

Vector normal del plano dado: (4,-2,4)

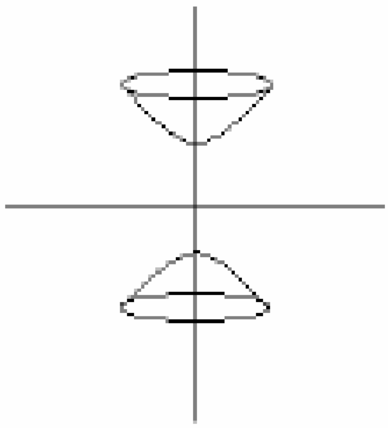
Condición: $(x_o, -2y_o, -4z_o) = \lambda(2,-1,2) \Rightarrow x_o = 2\lambda$
$y_o = \lambda/2$
$z_o = -\lambda/2$.

Y como Po es $(2\lambda)^2 - 2(\lambda/2)^2 - 4(-\lambda/2)^2 = 16 \therefore \lambda = \pm\frac{4\sqrt{2}}{\sqrt{5}}$

Luego los puntos son:

$$Po\left(\frac{8\sqrt{2}}{\sqrt{5}}, \frac{16\sqrt{2}}{\sqrt{5}}, -\frac{32\sqrt{2}}{\sqrt{5}}\right)$$

$$P1\left(-\frac{8\sqrt{2}}{\sqrt{5}}, -\frac{16\sqrt{2}}{\sqrt{5}}, \frac{32\sqrt{2}}{\sqrt{5}}\right)$$



4.

$a)$

$$V=\iint_D \sqrt{4a^2-(x^2+y^2)}\,dA_{xy}=\int_0^{\pi/4}\int_0^a \sqrt{4a^2-\rho^2}\,\rho d$$

$$V=-\frac{1}{2}\int_0^{\pi/4}\frac{2}{3}(4a^2-\rho^2)^{3/2}\Big|_0^a d\theta=\frac{1}{2}(8a^3-(3a^2)^{3/2}$$

$$V=\frac{\pi}{8}(8-3^{3/2})a^3$$

b)

$$A(s)=\iint_D \sqrt{1+z_x^2+z_y^2}\,dA_{xy}=\iint_D \sqrt{1+\frac{x^2+y^2}{4a^2-(x^2+y^2)}}$$

$$A(s)=2a\iint_D \frac{dA}{\sqrt{4a^2-(x^2+y^2)}}=2a\int_0^{\pi/4}\int_0^a \frac{\rho d\,\rho d\theta}{\sqrt{4a^2\rho^2}}$$

$$A(s)=-a\int_0^{\pi/4} 2(4a^2-\rho^2)^{1/2}\Big|_0^a d\theta=(4a^2-a^2\sqrt{3})\pi/4$$